

Introduction to Value Tree Analysis

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About the introduction

- This is a brief introduction to multiple criteria decision analysis and specifically to value tree analysis
- After reading the material you should know
 - basic concepts of value tree analysis
 - how to construct a value tree
 - how to use the Web-HIPRE software in simple decision making problems to support your decision



Basic concepts

Objective

- is a statement of something that one desires to achieve
- for example; “more wealth”

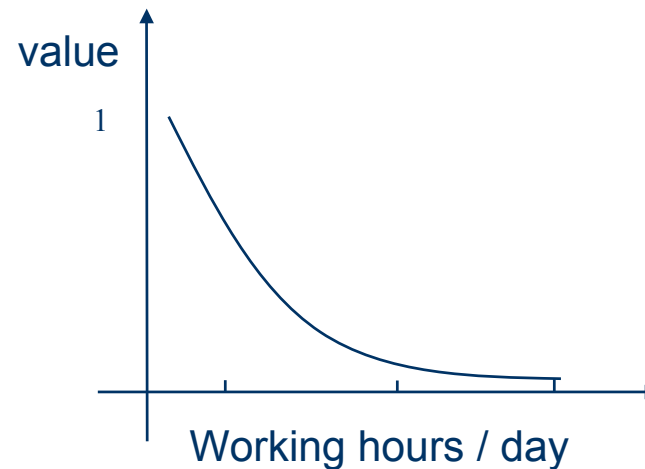
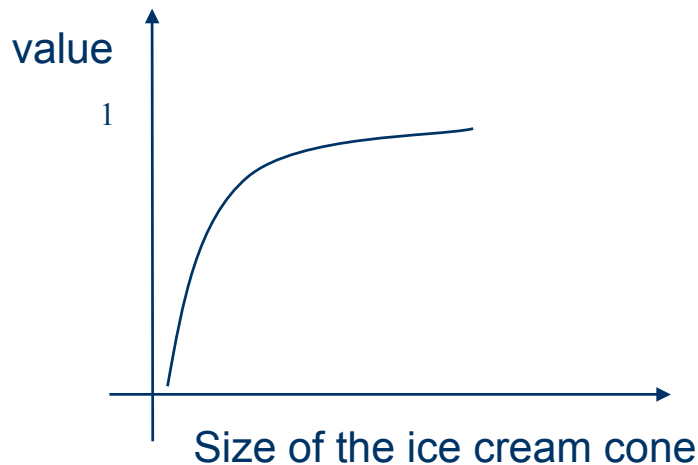
Attribute

- indicates the level to which an objective is achieved in a given decision alternative
- for example by selecting a certain job offer you may get 3000 €/month



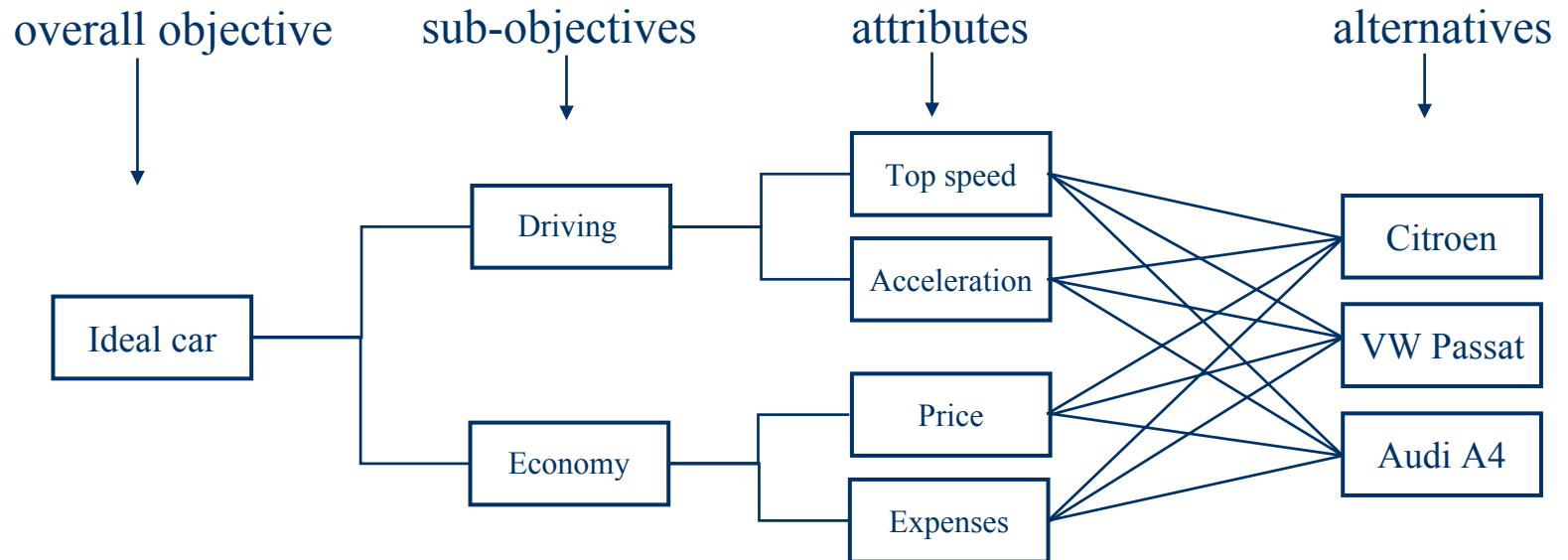
Value function

- Value function $v(x)$ assigns a number i.e. *value* to each attribute level x .
- Value describes subjective desirability of the corresponding attribute level.
- For example:



Value tree

In a value tree objectives are organised hierarchically



- Each objective is defined by sub-objectives or attributes
- There can be several layers of objectives
- Attributes are added under the lowest level of objectives
- Decision alternatives are connected to the attributes

Phases of value tree analysis

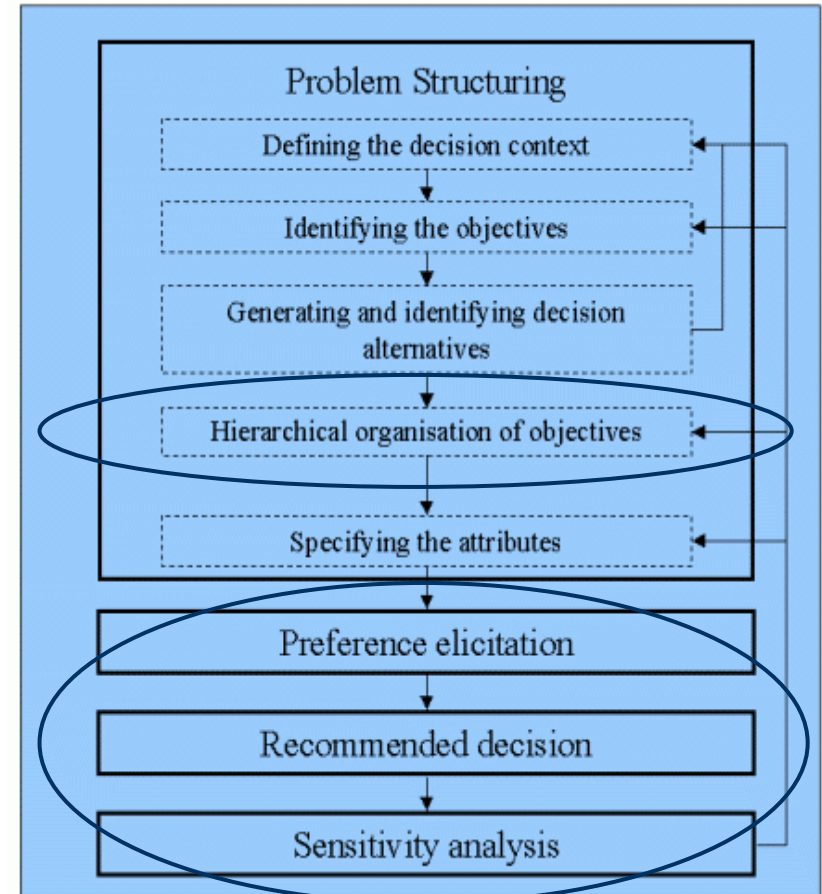
The aim of the Problem structuring is to create a better understanding of the problem

Decision context is a setting in which the decision occurs

In Preference elicitation DM's preferences over a set of objectives is estimated and measured

The aim of the Sensitivity analysis is to explore how changes in the model influence the recommended decision

Note: Only the highlighted parts are covered in this mini intro



A job selection problem

Assume that you have four job offers to choose between;

- 1) a place as a **researcher in a governmental research institute**
- 2) a place as a **consultant in a multinational consulting firm**
- 3) a place as a **decision analyst in a large domestic firm**
- 4) a place in a **small IT firm**

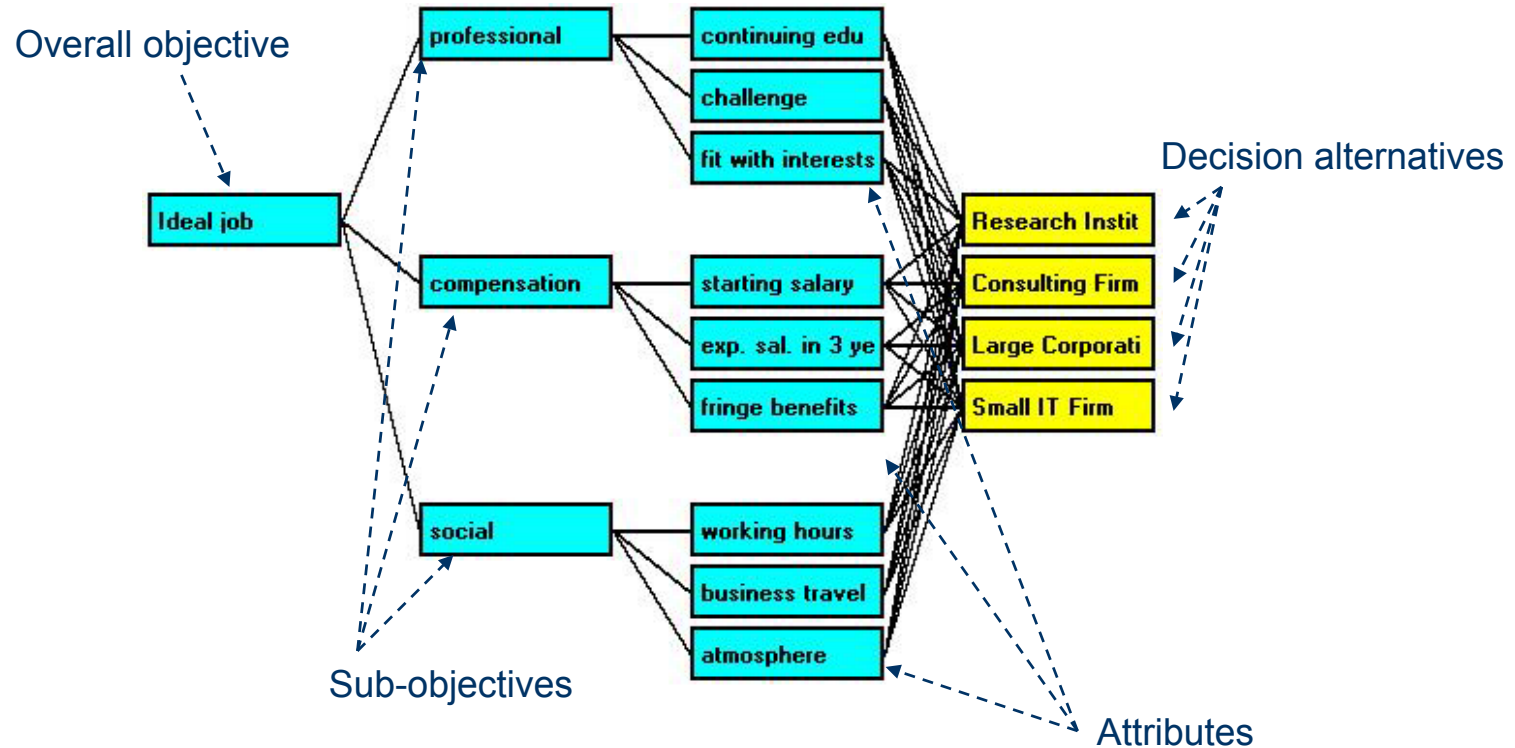


Hierarchical organisation of objectives

- 1) Identify the overall objective.
- 2) Clarify its meaning with more specific sub-objectives. Add the sub-objectives to the next level of the hierarchy.
- 3) Continue recursively until an attribute can be associated with each lowest level objective.
- 4) Add the decision alternatives to the hierarchy and link them to the attributes.
- 5) Iterate the steps 1- 4, until you are satisfied with the structure.



The objectives hierarchy for the job selection problem



Video Clip: Structuring a value tree in Web-HIPRE
[with sound](#) (.avi 3.3MB)
[no sound](#) (.avi 970KB)
[animation](#) (.gif 475KB)

Consequences

Attribute	Research Institute	Consulting Firm	Large Corporation	Small IT Firm
continuing education	3	3	1	2
starting salary/€	1900	2700	2200	2300
expected salary in 3 years/€	2500	3500	2800	3000
hours / week	37.5	55	40	42.5
atmosphere	3.2	2.5	3.7	4.5
travelling days / year	20	160	100	30

Video Clip: Entering the consequences of the alternatives in Web-HIPRE
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[animation](#) (.gif 165 KB)

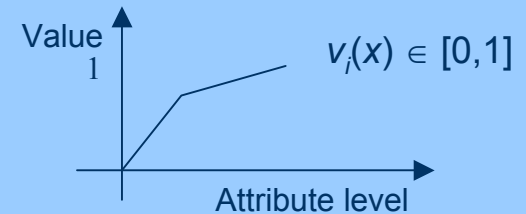


Preference elicitation: an overview

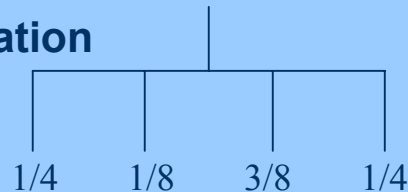
The aim is to measure DM's preferences on each objective.

Value elicitation

First, single attribute value functions v_i are determined for all attributes X_i .



Weight elicitation



Second, the relative weights of the attributes w_i are determined.

Finally, the total value of an alternative a with consequences $X_i(a)=x_i$ ($i=1..n$) is calculated as

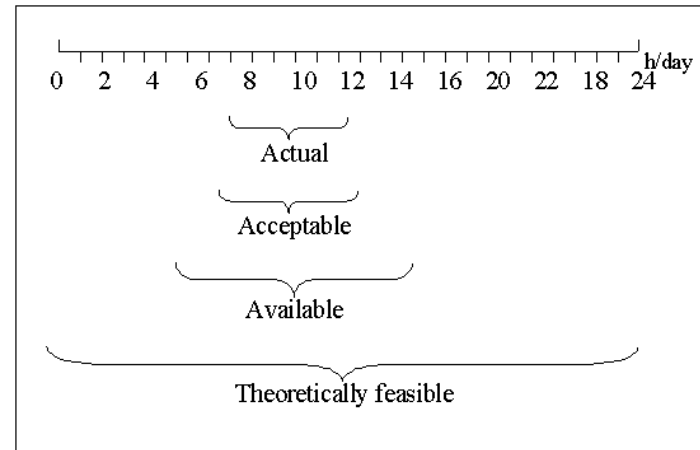
$$V(x_1, x_2, \dots, x_n) = \sum_{i=1}^n w_i v_i(x_i)$$



Single attribute value function elicitation in brief

1) Set attribute ranges

- All alternatives should be within the range.
- Large range makes it difficult to discriminate between alternatives.
- New alternatives may lay outside the range if it is too small.



Possible ranges for the “working hours/d” attribute

2) Estimate value functions for attributes

- **Assessing the form of value function**
- **Direct rating**
- Bisection
- Difference standard sequence
- Category estimation
- Ratio estimation
- AHP

Note:

Methods used in this case are shown in bold



Setting attributes' ranges

- No new job offers expected
- Analysis is used to compare only the existing alternatives

→ small ranges are most appropriate

Attribute	Research Institute	Consulting Firm	Large Corporation	Small IT Firm	Range
continuing education	3	3	1	2	1 - 3
starting salary/€	1900	2700	2200	2300	1900 - 2300
expected salary in 3 years/€	2500	3500	2800	3000	2500 - 3500
hours / week	37.5	55	40	42.5	37.5 - 55
atmosphere	3.2	2.5	3.7	4.5	2.5 - 4.5
travelling days / year	20	160	100	30	20 - 160



Assessing the form of value function

Is the value function

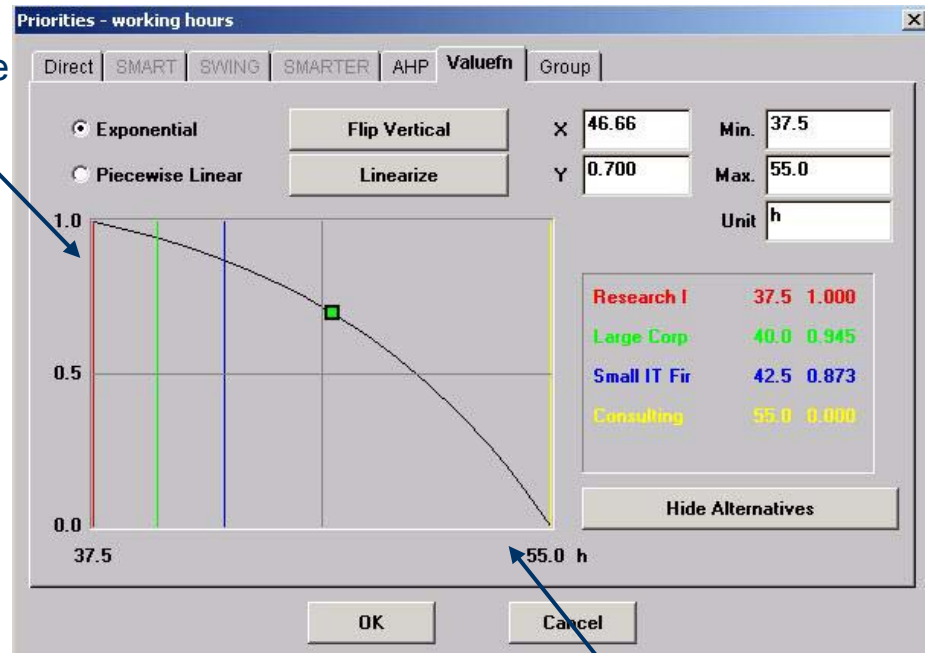
- increasing or decreasing?
- linear?

Is an increase at the end of the attribute scale more important than a same sized increase at the beginning of the scale?

You can use *Bisection method* to ease the assessment.

More about [the Bisection method](#) (optional)

Value scale



Attribute level scale

In the following video clip the Bisection method is used to estimate a point from the value curve. Web-HIPRE uses exponential approximation to estimate the rest of the value function.

Video Clip: Assessing the form of the value function with bisection method in Web-HIPRE
[with sound](#) (.avi 1.69 MB)
[no sound](#) (.avi 303 KB)
[animation](#) (.gif 180 KB)



Direct rating

- 1) Rank the alternatives
- 2) Give 100 points to the best alternative
- 3) Give 0 points to the worst alternative
- 4) Rate the remaining alternatives between 0 and 100

Note that direct rating:

- is most appropriate when the performance levels of an attribute can be judged only with subjective measures
- can be used also for weight elicitation

Video Clip: Using direct rating in Web-HIPRE

[with sound](#) (.avi 1.17 MB)

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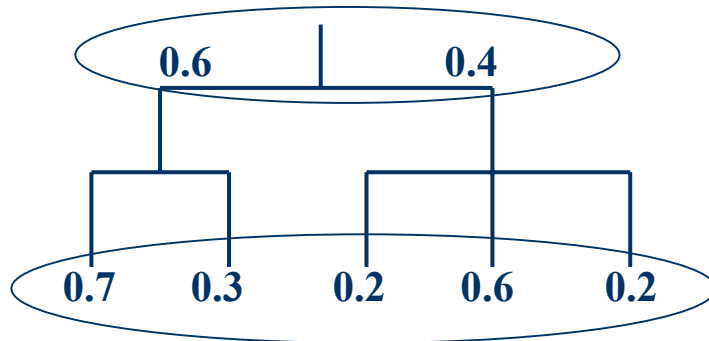
[animation](#) (.gif 142 KB)



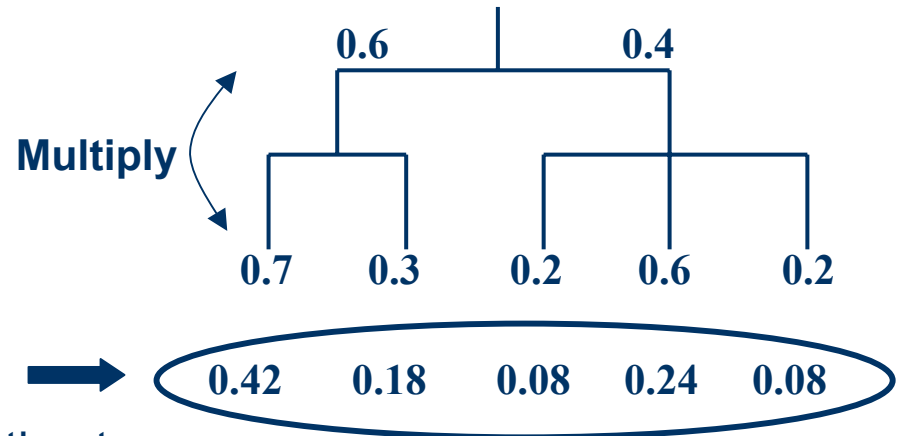
About weight elicitation

In the Job selection case hierarchical weighting is used.

1) Weights are defined for each hierarchical level...



2) ...and multiplied down to get the final lower level weights.



To improve the quality of weight estimates

- use several weight elicitation methods
- iterate until satisfactory weights are reached

In the following the use of different weight elicitation methods is presented...



SMART

- 1) Assign 10 points to the least important attribute (objective)

$$w_{\text{least}} = 10$$

- 2) Compare other attributes with x_{least} and weigh them accordingly

$$w_i > 10, i \neq \text{least}$$

- 3) Normalise the weights

$$w'_k = w_k / (\sum_i w_i), i = 1 \dots n, n = \text{number of attributes (sub-objectives)}$$

Video Clip: Using SMART in Web-HIPRE

[with sound](#) (.avi 1.12 MB)

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[animation](#) (.gif 133 KB)



AHP

1) Compare each pair of

- sub-objectives or attributes under an objective

2) Store *preference ratios* in a *comparison matrix*

- for every i and j , give r_{ij} , the ratio of importance between the i th and j th objective (or attribute, or alternative)

- Assign $A(i,j) = r_{ij}$
- $$A = \begin{pmatrix} r_{11} & \dots & r_{1n} \\ \dots & \dots & \dots \\ r_{n1} & \dots & r_{nn} \end{pmatrix}$$

3) Check *the consistency measure* (CM)

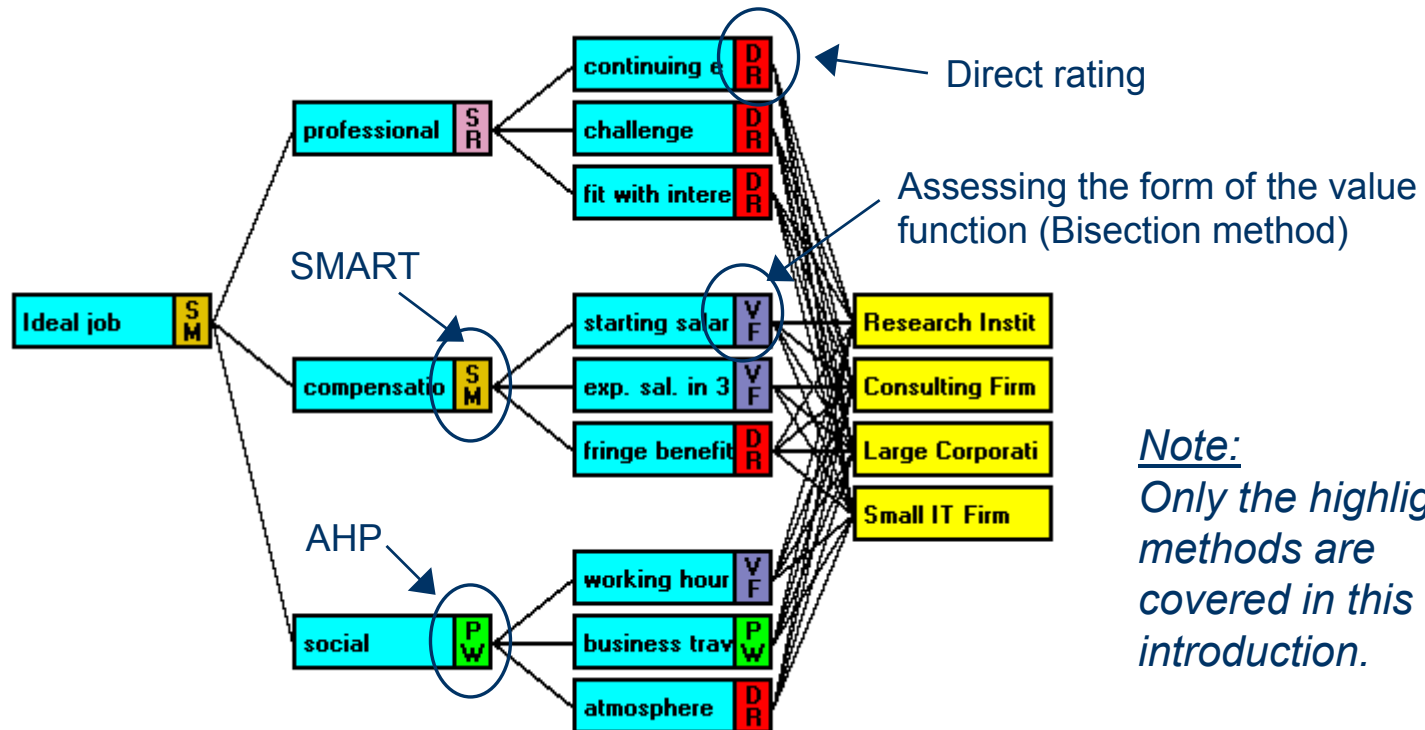
- If $CM > 0.20$ identify and eliminate inconsistencies in preference statements

Video Clip: Using AHP in Web-HIPRE
[with sound](#) (.avi 1.97 MB)
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[animation](#) (.gif 204 KB)



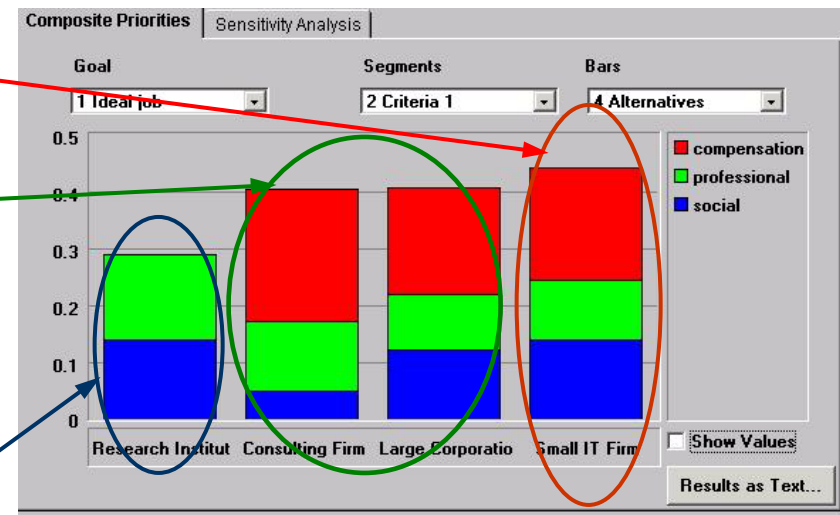
Used preference elicitation methods

- The job selection value tree with used preference elicitation methods shown in Web-HIPRE:



Recommended decision

- **Small IT firm is the recommended alternative with the highest total value (0.442)**
- Large corporation and consulting firm options are almost equally preferred (total values 0.407 and 0.405 respectively)
- Research Institute is clearly the least preferred alternative (total value of 0.290)



Solution of the job selection problem in Web-HIPRE. Only first-level objectives are shown.

Video Clip: Viewing the results in Web-HIPRE

[with sound](#) (.avi 1.58 MB)

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[animation](#) (.gif 213 KB)



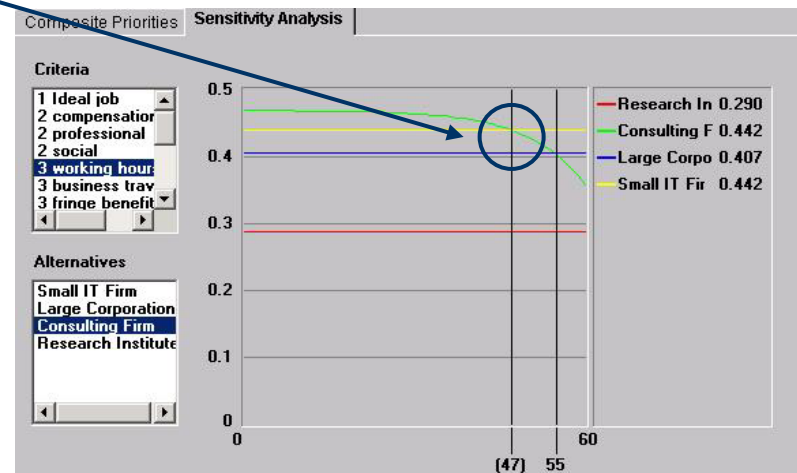
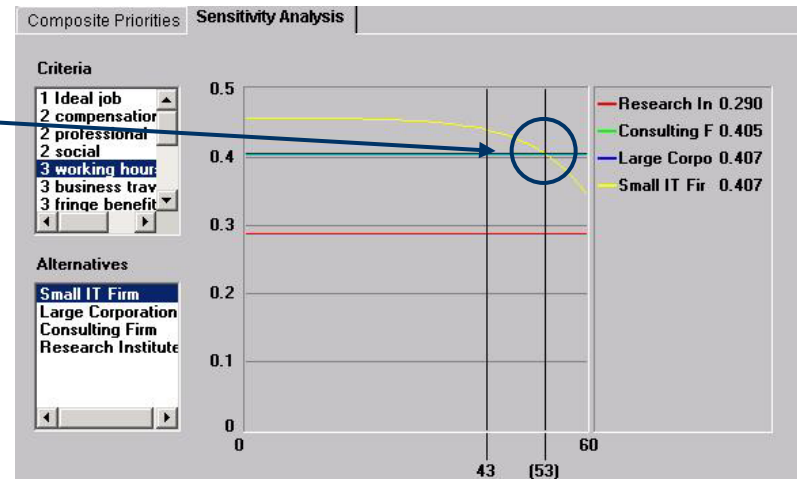
One-way sensitivity analysis

- What happens to the solution of the job selection problem if one of the parameters affecting the solution changes? What if, for example the working hours in the IT firm alternative increase to 50 h/week or the salary in the Research Institute rises to 2900 euros/month?
- In other words, how sensitive our solution is to changes in the objective weights, single attribute value functions or attribute ratings
- In ***one-way sensitivity analysis*** one parameter is varied at time
- Total values of decision alternatives are drawn as a function of the variable under consideration
- Next, we apply one-way sensitivity analysis to the job selection case



Changes in “working hours” attribute

- If working hours in the IT firm rise to 53 h/week or over and nothing else in the model changes, Large Corporation becomes the most preferred alternative
- If working hours in the Consulting firm were 47 h/week or less instead of the current 55 h/week, it would be considered the best alternative



Changes in “working hours” attribute

- Changes in the weekly working hours in Large corporation’s job offer would not affect the recommended solution even if they decreased to zero. The ranking order of the other alternatives would change though.
- Changes in the weekly working hours in the Research Institute’s job offer don’t have any effect on the solution or on the preference order of rest of the alternatives.

Video Clip: Sensitivity analysis in Web-HIPRE
[with sound](#) (.avi 1.60 MB)
[no sound](#) (.avi 326 KB)
[animation](#) (.gif 239 KB)

